Steady Flow to a Well

Revised: 3/28/2012 jcw

Purpose:

This laboratory exercise in intended to investigate steady confined and unconfined flow in an aquifer to a well. Many cities, residences, and farms rely on wells for water supply. Pumping of water from a well lowers the hydraulic head near the well causing surrounding water to move towards the well (down the energy gradient). The rapidity of water movement through the aquifer towards the well often sets the limit for how rapidly the well can be pumped. Other factors include head loss passing through the well screen, head loss flowing from the pump to the ground surface, and pump capacity. Where is the pump located in most wells? Why? Does the pumping energy depend upon the depth of the well or the difference in head between the pump and the surface? Why?

Background:

An unconfined aquifer is one that ends with a water table on top such as the sand in the hydrology unit. Unconfined, steady state, groundwater flow depends upon the gradient in hydraulic head (energy per unit weight), hydraulic conductivity, and the thickness of the flow unit. If datum is defined to be the bottom of the unit, and flow is predominantly in the horizontal direction, then the hydraulic head is equal to the depth of water. Idealized horizontal flow is referred to as the Dupuit assumption. For radial, one dimensional flow to a well in an infinite domain, the governing equation is:

$$Q = \pi K \frac{(h_2^2 - h_1^2)}{\ln\left(\frac{r_2}{r_1}\right)} \tag{1}$$

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where:

Q	=	flow rate from well (m^3/s)
K	=	hydraulic conductivity (m/s) = $\kappa \frac{\gamma}{\mu}$
h	=	saturated thickness of aquifer at location 1 and 2 (m)

r = radial distance from well at location 1 and 2 (m)

Where location 1 is closer to the well than location 2.

Equation 1 can be derived by considering steady flow through a cylinder located at distance r from the well. Flow through the cylinder by Darcy's Law is:

$$Q = 2\pi r h K \frac{dh}{dr} \qquad (2)$$

Separating variables and integrating gives:

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi K}{Q} \int_{h_1}^{h_2} h\,dh \qquad (3)$$

giving Equation 1 as the solution. For a confined aquifer the equation is: For example, water may flow between a river and a canal or between lakes. In a confined aquifer the solution is:

$$Q = 2\pi K b \frac{(h_2 - h_1)}{\ln(r_2 / r_1)} \qquad (4)$$

The derivation is left up to the student as an exercise.

Although an infinite domain is assumed in deriving Equation 1, our hydrology system is not infinite in size. The no flow boundaries on two sides tend to increase the drawdown in wells while the fixed head areas decrease the drawdown.

The derived equations gives the change in head (h_2-h_1), or drawdown between any two wells. In order to facilitate use of this equation it is useful to define the concept of radius of influence. The radius of influence is the distance from the well where the well no longer has any influence on the hydraulic head. In the real world this is usually the distance where recharge to the ground resupplies the pumped water. In the laboratory watershed, we create a radius of influence by setting the hydraulic head at a finite distance from the well.

Methods:

Break into groups and perform separate experiments for each group. Perform the unconfined test first. Turn on the rain for a short period to saturate the sand, but turn off the rain for the experiment. Make the surface of the sand relatively flat. Dig a trench in a circle around the well to fix the hydraulic head at that distance and connect the circle to the water supply. Make the circle as large as possible (diameter of 1 m). Raise the water level at the ends of the hydrology unit as high as possible by blocking up the usual water exit point. Set up the system to provide resupply water from the hidden tubes at each end of the hydrology unit. Carefully invert the peizometer columns and shake out the air from the tubes to allow for accurate readings. Open the upper well and make sure the lower well is closed. Measure the water table height at the piezometers after steady state is reached as well as the water level in the trench by using a meter stick. Note that the datum on the piezometers needs to match the bottom of the sand tank for the unconfined aquifer calculations. This is because the h represents both the head and the saturated thickness. Correct to proper datum by subtracting 1.5 cm from each piezometer reading. Measure the flow from the well at steady state. There should be at least 8 (preferably 12) piezometers within the circle around the well. What is your datum? What is the radius (r_2) to the fixed head? Did you measure the distance from the well to each piezometer?

Repeat the experiment after placing a circular sheet of plastic under the sand to simulate a confining layer. Cover the sheet with sand and flood water over the top of it. The plastic sheet should be placed 11 cm from the sides and 36 cm from the end in order to be centered on the well. Be sure to measure the thickness of your confined aquifer. The edge of the sheet is the radius of influence (r_2 and h_2).

Analysis

Using Equations 1 and 4 estimate the hydraulic conductivity – a constant specific to the sand in the tank that should be the same for each measurement. Give the hydraulic conductivity as a mean, standard deviation, and 95% confidence interval based upon your multiple flow measurements. Any variations from a contstant value indicate experimental errors and uncertainty. You should have at least 8 measurement points for the unconfined and 8 for the confined experiment. Plot the measured and calculated drawdown (h_2-h_1) as a function of radial distance from the well for both confined and unconfined aquifers using the mean hydraulic conductivity calculated above. Where do you find the greatest discrepancy between measured and calculated heads?

The governing equations relate the hydraulic head at any two locations to the flow rate. We have one equation for unconfined aquifers and a different one for confined aquifers. In our experiment

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the hydraulic head is measured at all the piezometers located inside the plastic circle and at the edge of the circle. h_2 is the head at the edge, h_1 is the head at any individual piezometer. Each piezometer is located a distance (r) from the well. Note that r is the same for several of the piezometers. Q, the flow from the well, is measured by timing flow into a cylindrical flask. The predicted drawdown is (h_i - h_1). Predicted and measured drawdowns can now be compared with each other at each piezometer using the average value for hydraulic conductivity calculated previously.

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